Data Mining

Association Analysis

Introduction to Data Mining, 2nd Edition

by

Tan, Steinbach, Karpatne, Kumar

Introduction

- Business enterprises accumulate large quantities of data from their day-to-day operations
  - *huge amounts of customer purchase data* collected daily at the checkout counters of grocery stores.

Market-Basket transactions

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
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- Analyze the data to learn about the **purchasing behaviour** of the customers.
- Valuable information can be used to support variety of **business-related applications**:
  - → ***Marketing promotions***
  - → ***Inventory managment***
  - → ***CRM***
  - → ***Pricing***

Association Rule Mining

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

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Example of Association Rules

{Diaper} → {Beer},
{Milk, Bread} → {Eggs,Coke},
{Beer, Bread} → {Milk},

Implication means co-occurrence, not causality!

Definition: Frequent Itemset

- **Itemset**
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items

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  - Frequency of occurrence of an itemset
  - E.g. σ({Milk, Bread,Diaper}) = 2

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  - Fraction of transactions that contain an itemset
  - E.g. s({Milk, Bread, Diaper}) = 2/5

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- **Support**
  - Fraction of transactions that contain an itemset
  - E.g. s({Milk, Bread, Diaper}) = 2/5
- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a ***minsup*** threshold

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Definition: Association Rule

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  - An implication expression of the form X→ Y, where X and Y are itemsets
  - Example:
    {Milk, Diaper}→{Beer}

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- **Rule Evaluation Metrics**
  - Support (s)
    - Fraction of transactions that contain both X and Y
  - Confidence (c)
    - Measures how often items in Y appear in transactions that contain X

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    - Fraction of transactions that contain both X and Y
  - Confidence (c)
    - Measures how often items in Y appear in transactions that contain X

Example:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \{\text{Beer}\}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|\text{T}|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

Association Rule Mining Task

- Given a set of transactions T, the goal of association rule mining is to find all rules having
  - support ≥ ***minsup*** threshold
  - confidence ≥ ***minconf*** threshold

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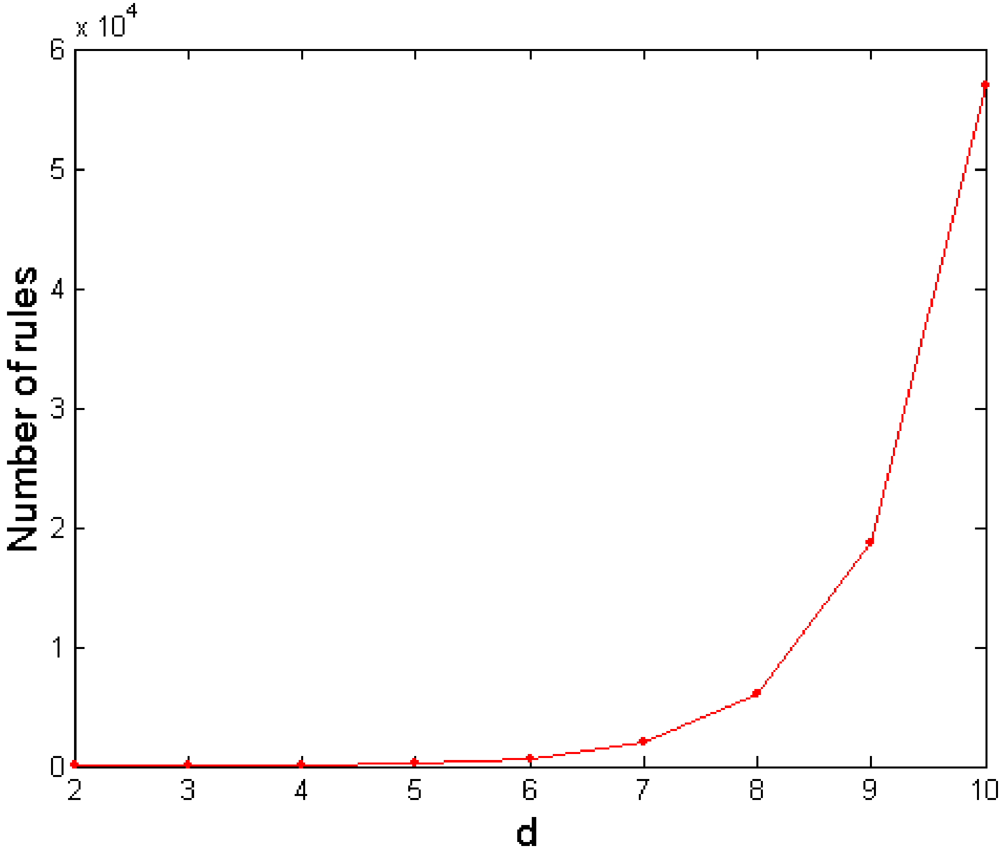
- Given a set of transactions T, the goal of association rule mining is to find all rules having
  - support ≥ ***minsup*** threshold
  - confidence ≥ ***minconf*** threshold

- Brute-force approach:
  - List all possible association rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the ***minsup*** and ***minconf*** thresholds

  ⇒ Computationally prohibitive!

Computational Complexity

- Given d unique items:
  - Total number of itemsets = 2^d
  - Total number of possible association rules:



$$R = \sum_{k=1}^{d-1} \left[\binom{d}{k} \times \sum_{j=1}^{d-k} \binom{d-k}{j} \right]$$

$$= 3^d - 2^{d+1} + 1$$

If d=6, R = 602 rules

Mining Association Rules

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1	Bread, Milk
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Example of Rules:

{Milk,Diaper} → {Beer} (s=0.4, c=0.67)
{Milk,Beer} → {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} → {Milk} (s=0.4, c=0.67)
{Beer} → {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} → {Milk,Beer} (s=0.4, c=0.5)
{Milk} → {Diaper,Beer} (s=0.4, c=0.5)

Mining Association Rules

Observations:

- All the rules are binary partitions of the same itemset:
  {Milk, Diaper, Beer}

- Rules originating from the same itemset have **identical support** but can have **different confidence**

- Thus, we may **decouple** the support and confidence requirements

Mining Association Rules

Divide the problem in 2 sub tasks:

1. Frequent Itemset Generation
   - Find all the itemsets where support >= ***minsup*** threshold
2. Rule Generation
   - From above set find all itemsets with high confidence. These are strong rules

- Frequent Itemset Generation is still computationally expensive than Rule Generation

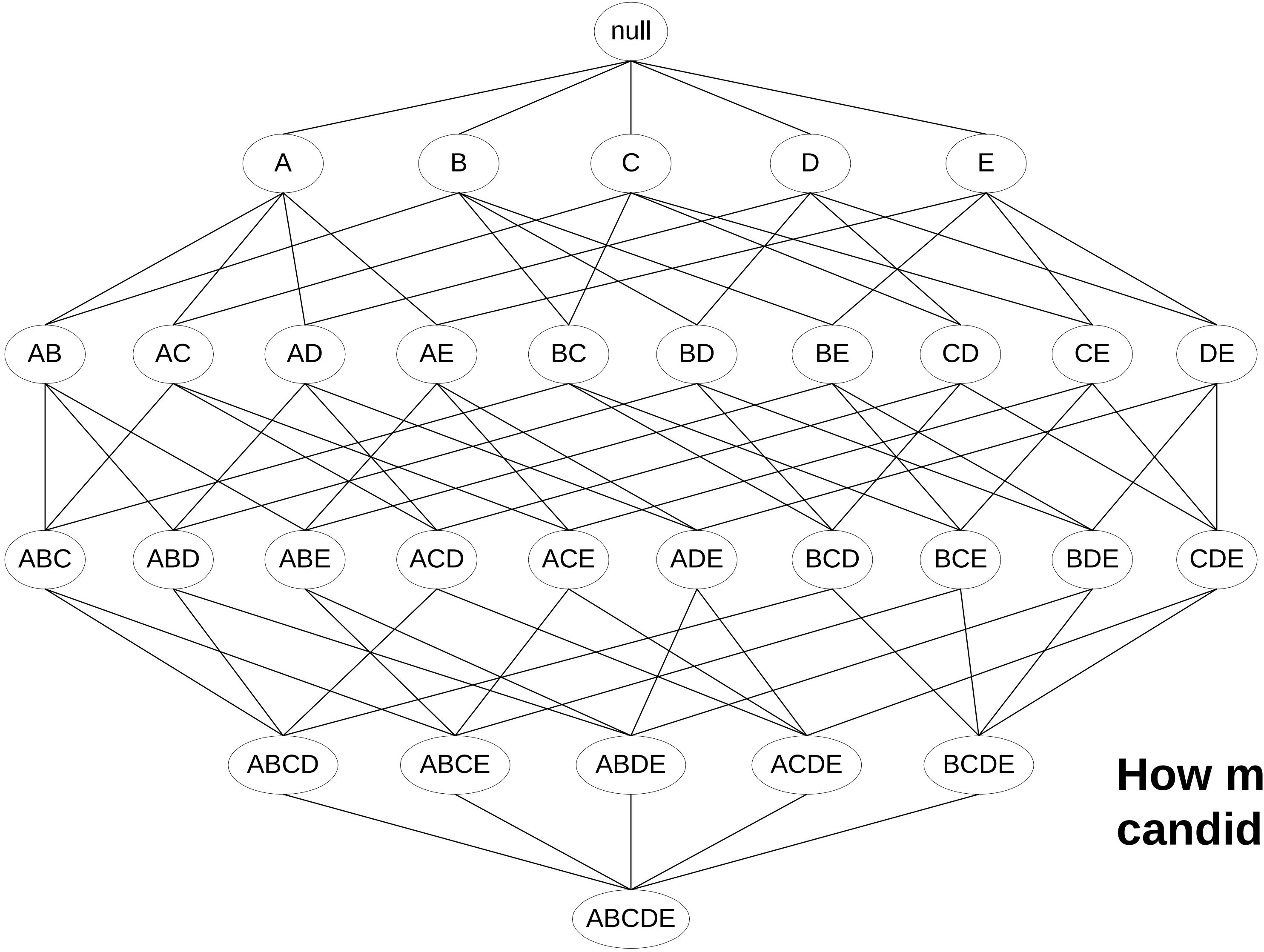
Frequent Itemset Generation

What are the list of possible itemsets?

For items, I = { a, b, c, d, e}

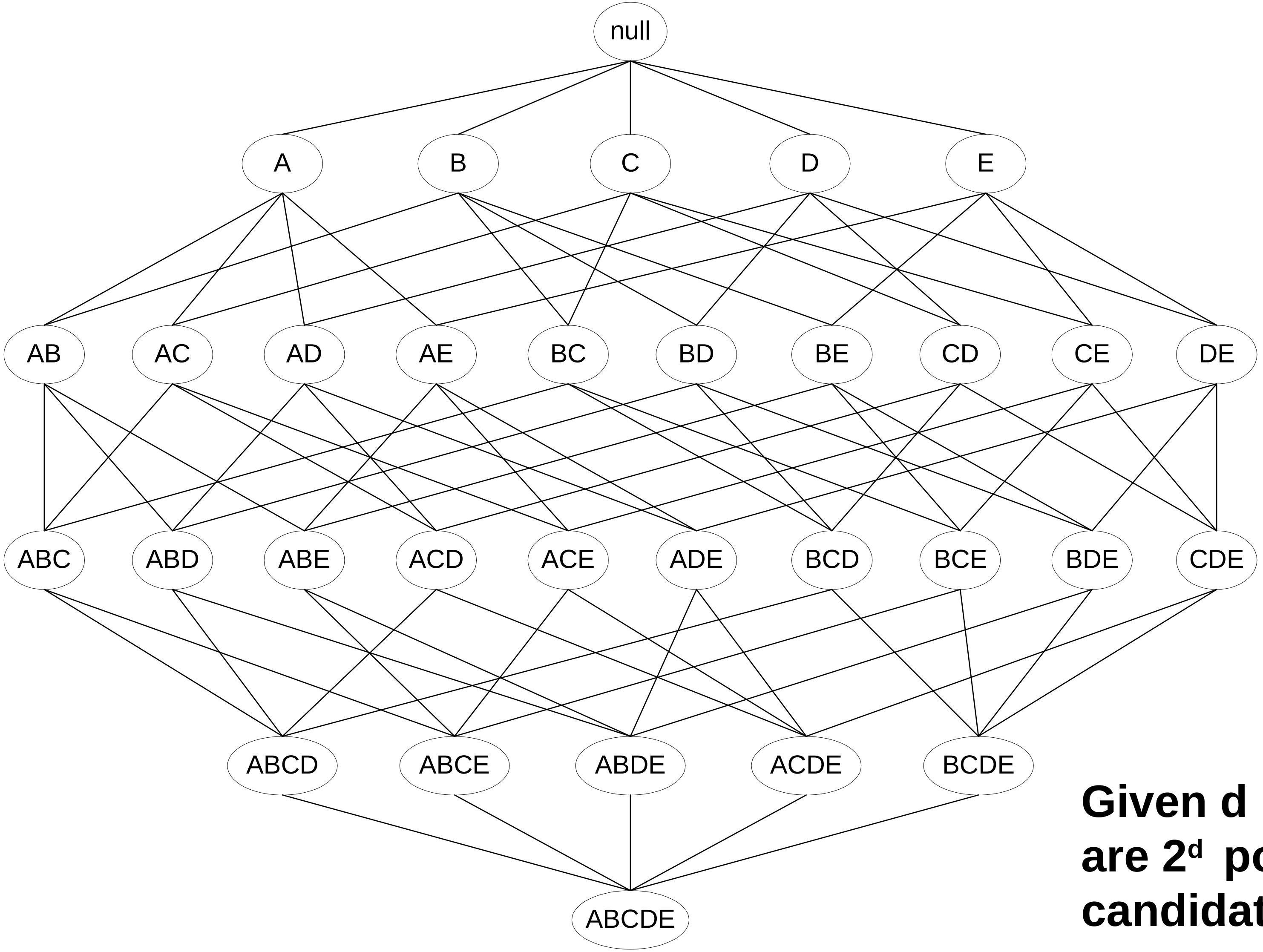
- Itemsets could be **a, b, c, d, e**
- **ab, ac, ad, ae**
- **bc, bd, be**
- **cd, ce**
- **de**
- **abc, abd, abe, acd, ace, and all such combinations**

Frequent Itemset Generation



How many possible candidate itemsets?

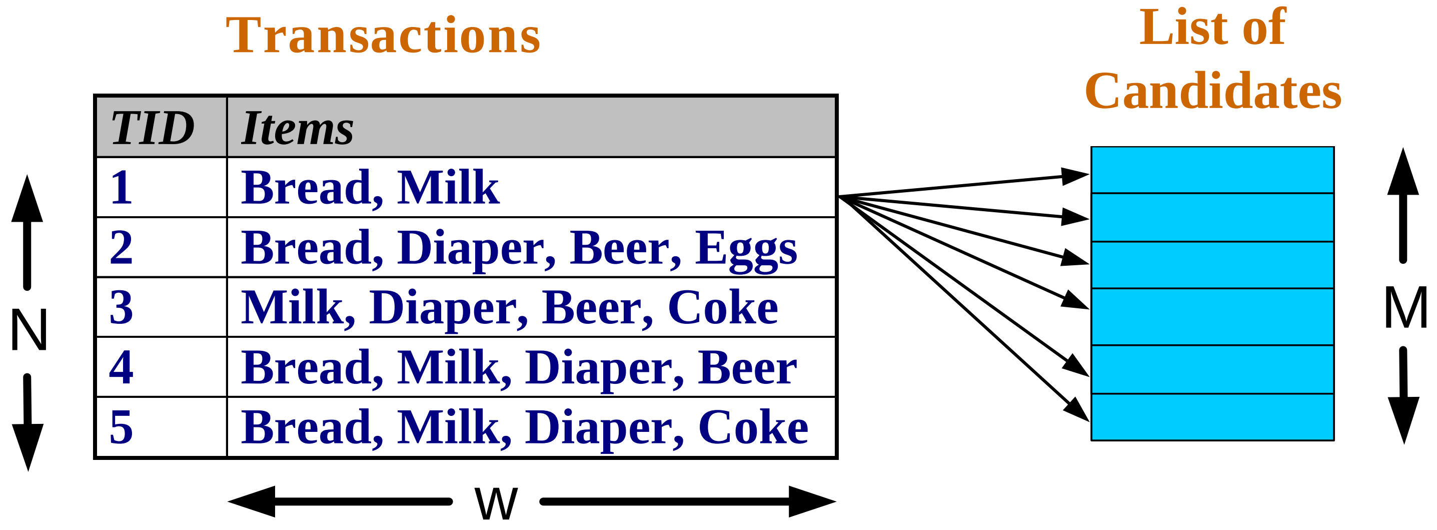
Frequent Itemset Generation



Given d items, there are 2^d possible candidate itemsets

Frequent Itemset Generation

- Brute-force approach:
  - Each itemset in the lattice is a candidate frequent itemset
  - Count the support of each candidate by scanning the database



Transactions

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
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List of Candidates

  - Match (compare) each transaction against every candidate
  - Complexity ~ O(NMw) => Expensive since M = 2^d-1 !!!

Frequent Itemset Generation Strategies

- Reduce the number of candidates (M)
  - Complete search: $M=2^d$
  - Use **pruning** techniques to **reduce M**

- Reduce the number of transactions (N)
  - Reduce size of N as the size of itemset increases
  - Used by DHP and vertical-based mining algorithms

- Reduce the number of comparisons (NM)
  - Use efficient data structures to store the candidates or transactions
  - No need to match every candidate against every transaction

The Apriori Principle

- Apriori principle:
  - If an itemset is frequent, then all of its subsets must also be frequent

- For example:
  - {Milk, Bread, Diapers} is a frequent itemset
    - means Milk, Bread, Diapers <u>support is high</u>
    - thus {Milk, Bread, Diapers} occur in many transaction

The Apriori Principle

- Apriori principle:

  If {Milk, Bread, Diapers} occurs in many transactions

  So does {Milk} , {Bread}, {Diapers}

  and

  {Milk, Bread}, {Milk, Diapers}, {Bread, Diapers}

The Apriori Principle

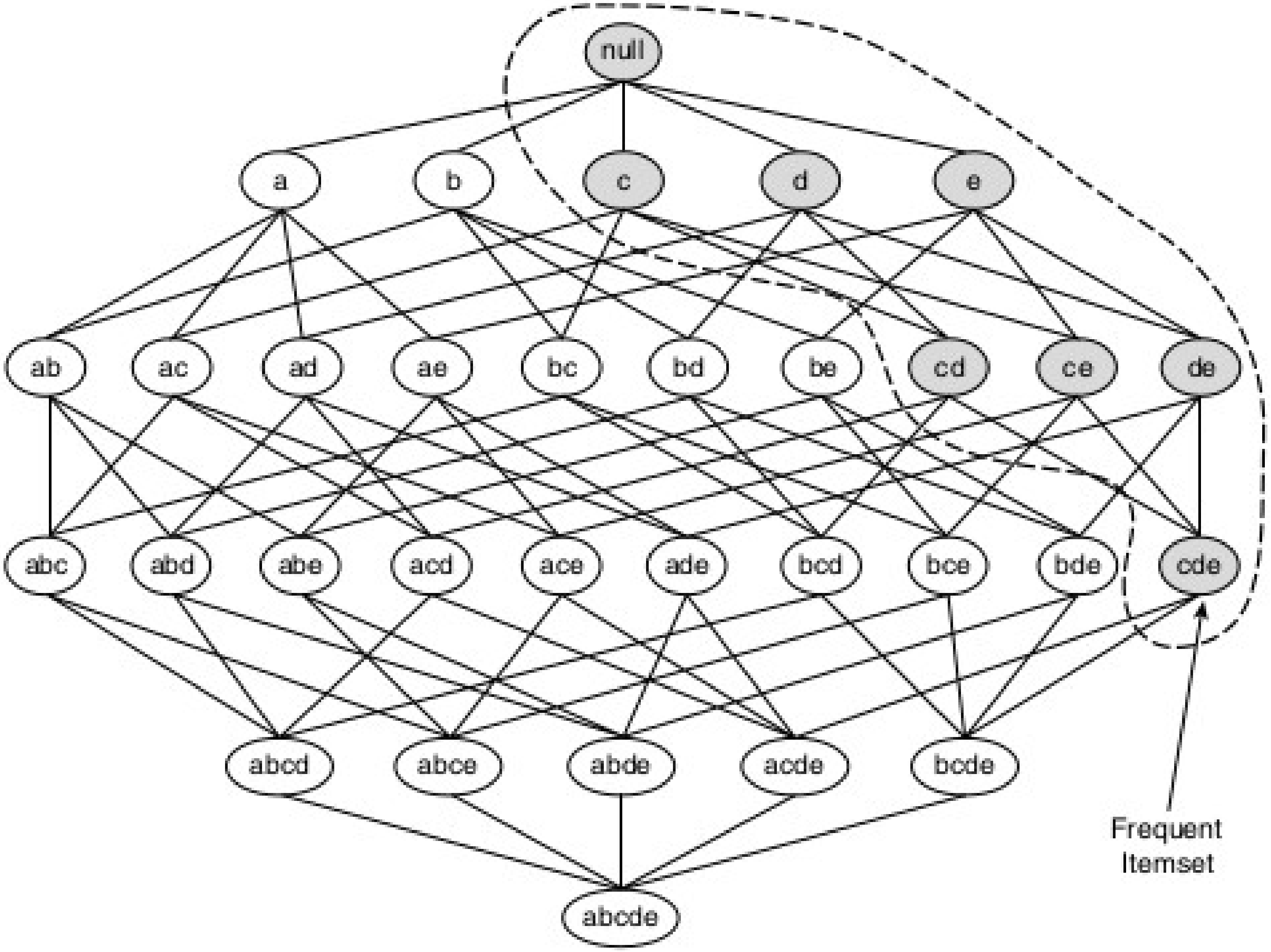

Figure 6.3. An illustration of the *Apriori* principle. If $\{c, d, e\}$ is frequent, then all subsets of this itemset are frequent.

The Apriori Principle

- Apriori principle:
  - If an itemset is infrequent, then all of its supersets must also be infrequent

- For example:
  - if {Coke} , {Jam} is infrequent itemset
    - means {Coke} , {Jam} support is low
    - thus {Coke, Jam} also don’t occur in many transaction

The Apriori Principle

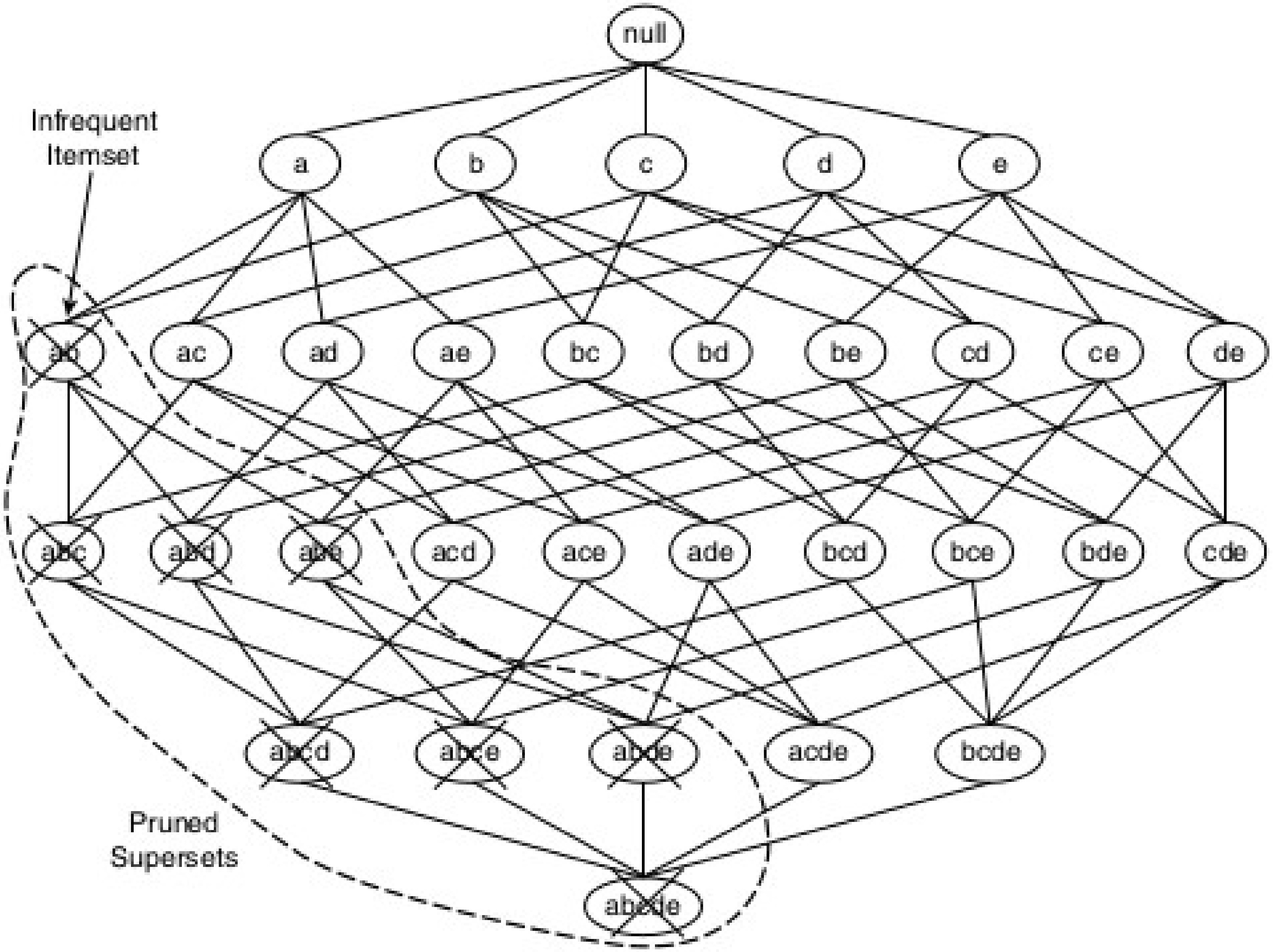

Figure 6.4. An illustration of support-based pruning. If $\{a, b\}$ is infrequent, then all supersets of $\{a, b\}$ are infrequent.

The Apriori Principle

- **Support-Based Pruning**.
  - Trimming the exponential search space based on the support measure.
  - Based on **Anti-Monotone Property**
    - Support of an itemset never exceeds the support of it’s subsets:
    - **I : set of items,**
    - **J = *2^I* (power set) // all possible combinations**
    - **S (support) is anti-monotone (or downward closed)**

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

    - Any anti-monotone measured can be used in mining

The Apriori Principle

- Monotonicity Property
  - I : set of items,
  - J = 2^{I} (power set) // all possible combinations
  - f is monotone (or upward closed)
    - $\forall X, Y \in J : (X \subseteq Y) \rightarrow f(X) \leq f(Y)$

Apriori Algorithm

- Uses Apriori Principle
- Support Based Pruning
- Control exponential growth of candidate itemset

Apriori Algorithm

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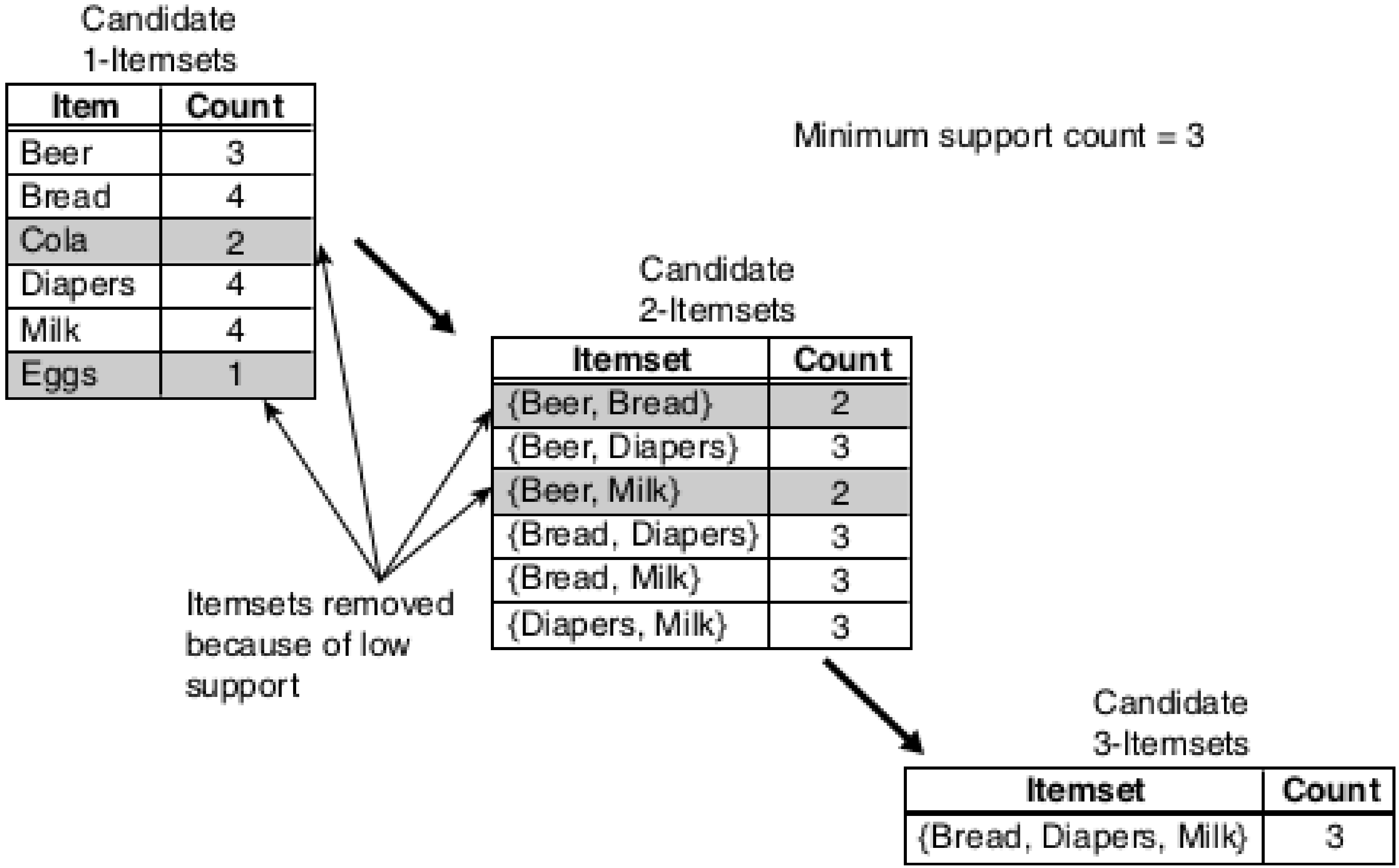


Candidate 1-Itemsets

Item	Count
Beer	3
Bread	4
Cola	2
Diapers	4
Milk	4
Eggs	1

Minimum support count = 3

Candidate 2-Itemsets

Itemset	Count
{Beer, Bread}	2
{Beer, Diapers}	3
{Beer, Milk}	2
{Bread, Diapers}	3
{Bread, Milk}	3
{Diapers, Milk}	3

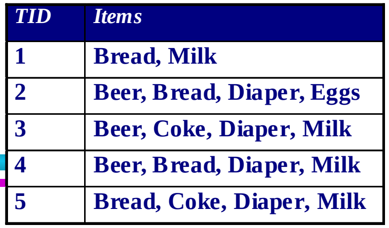
Itemsets removed because of low support

Candidate 3-Itemsets

Itemset	Count
{Bread, Diapers, Milk}	3

Figure 6.5. Illustration of frequent itemset generation using the *Apriori* algorithm.

Apriori Algorithm



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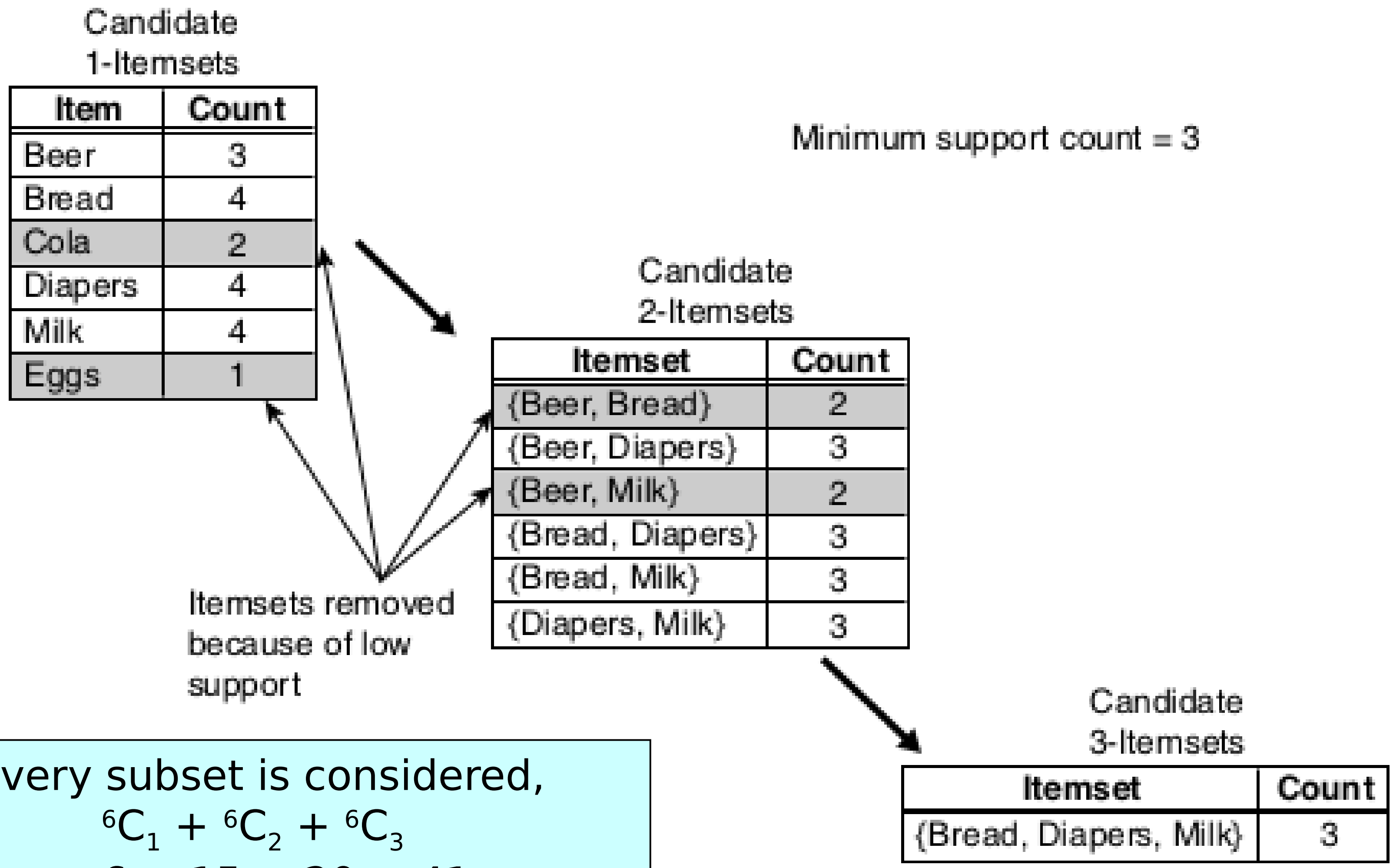

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Candidate 2-Itemsets

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{Beer, Milk}	2
{Bread, Diapers}	3
{Bread, Milk}	3
{Diapers, Milk}	3

Itemsets removed because of low support

Candidate 3-Itemsets

Itemset	Count
{Bread, Diapers, Milk}	3

mset generation using the *Apriori* algorithm.

If every subset is considered,

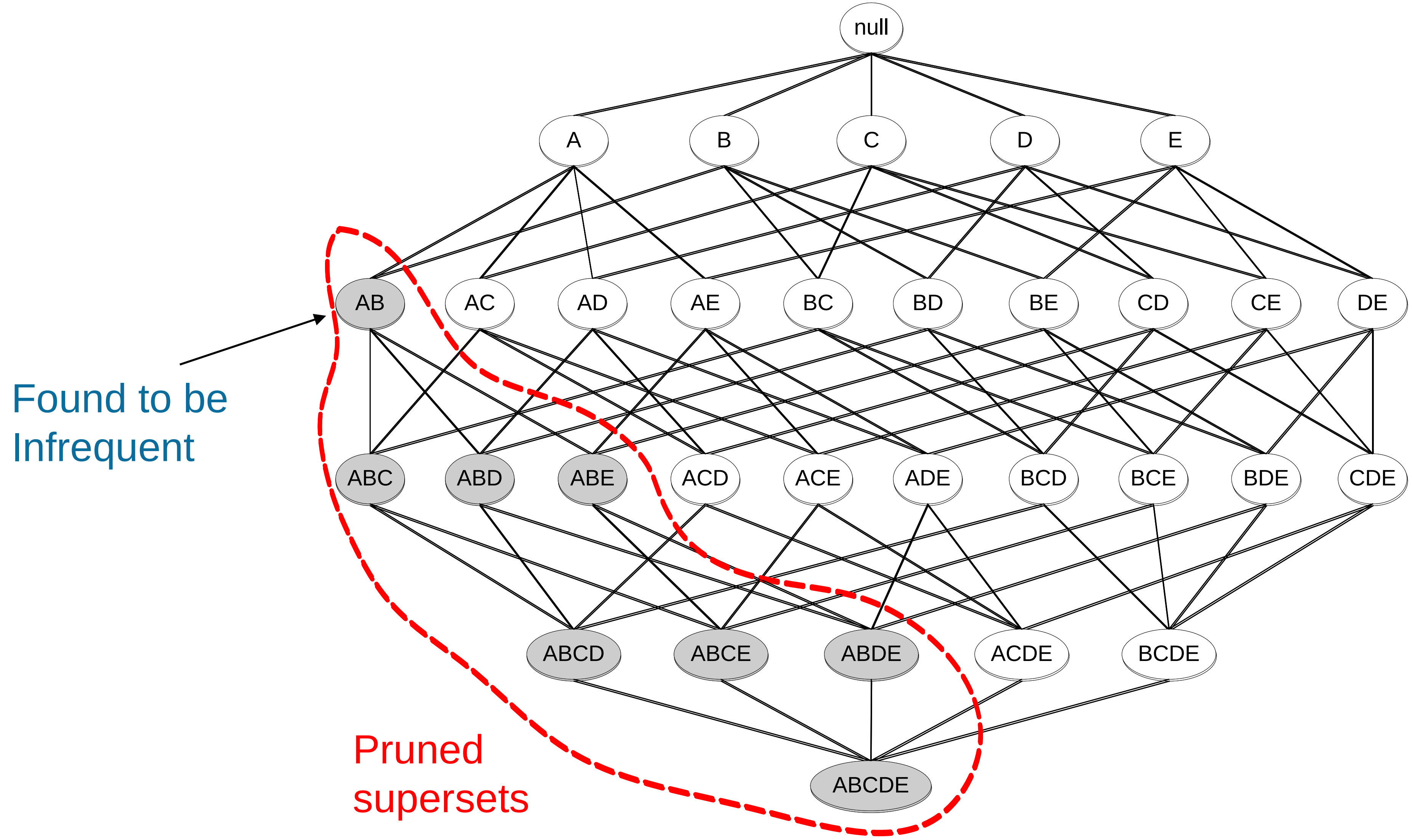
$^6C_1 + {}^6C_2 + {}^6C_3$

6 + 15 + 20 = 41

With support-based pruning,

6 + 6 + 1 = 13

Illustrating Apriori Principle



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Items (1-itemsets)

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

Minimum Support = 3

If every subset is considered,

$${}^6C_1 + {}^6C_2 + {}^6C_3$$

$$6 + 15 + 20 = 41$$

With support-based pruning,

$$6 + 6 + 4 = 16$$

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Items (1-itemsets)

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

Itemset
{Bread,Milk}
{Bread, Beer }
{Bread,Diaper}
{Beer, Milk}
{Diaper, Milk}
{Beer,Diaper}

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

If every subset is considered,

$${}^6C_1 + {}^6C_2 + {}^6C_3$$

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Itemset	Count
{Bread,Milk}	**3**
{Beer, Bread}	**2**
{Bread,Diaper}	**3**
{Beer,Milk}	**2**
{Diaper,Milk}	**3**
{Beer,Diaper}	**3**

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

If every subset is considered,

$${}^6C_1 + {}^6C_2 + {}^6C_3$$

$$6 + 15 + 20 = 41$$

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(No need to generate candidates involving Coke or Eggs)

Itemset	Count
{Bread,Milk}	**3**
{Bread,Beer}	**2**
{Bread,Diaper}	**3**
{Milk,Beer}	**2**
{Milk,Diaper}	**3**
{Beer,Diaper}	**3**

Minimum Support = 3

Triplets (3-itemsets)

Itemset
{ Beer, Diaper, Milk}
{ Beer,Bread,Diaper}
{Bread,Diaper,Milk}
{ Beer, Bread, Milk}

If every subset is considered,

$^6C_1 + {}^6C_2 + {}^6C_3$

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Beer	**3**
Diaper	**4**
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Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Itemset	Count
{Bread,Milk}	**3**
{Bread,Beer}	**2**
{Bread,Diaper}	**3**
{Milk,Beer}	**2**
{Milk,Diaper}	**3**
{Beer,Diaper}	**3**

Minimum Support = 3

Triplets (3-itemsets)

Itemset	Count
{ Beer, Diaper, Milk}	**2**
{ Beer,Bread, Diaper}	**2**
{Bread, Diaper, Milk}	**2**
{Beer, Bread, Milk}	**1**

If every subset is considered,

$${}^6C_1 + {}^6C_2 + {}^6C_3$$

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Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

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Minimum Support = 3

Triplets (3-itemsets)

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{ Beer,Bread, Diaper}	**2**
{Bread, Diaper, Milk}	**2**
{Beer, Bread, Milk}	**1**

If every subset is considered,

$$^6C_1 + {}^6C_2 + {}^6C_3$$

$$6 + 15 + 20 = 41$$

With support-based pruning,

$$6 + 6 + 4 = 16$$

$$6 + 6 + 1 = 13$$

Apriori Algorithm

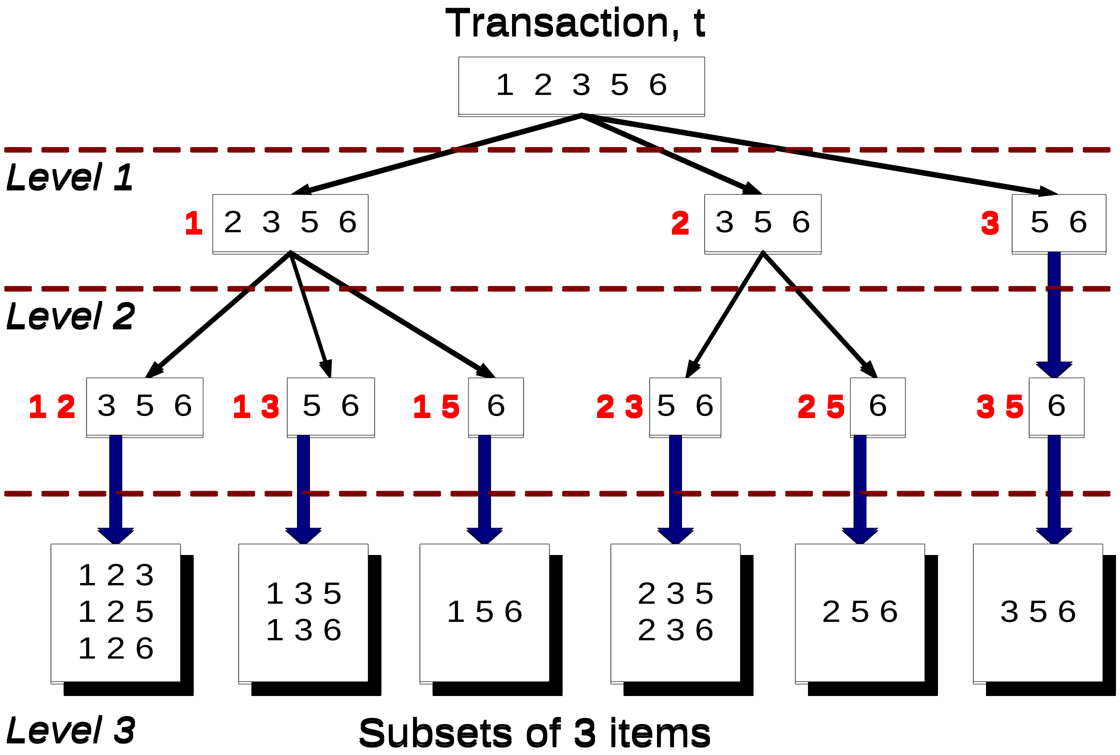
- Method:
  - Let k=1
  - Generate frequent itemsets of length 1
  - Repeat until no new frequent itemsets are identified
    - Generate length(k+1) candidate itemsets from length k frequent itemsets
    - Prune candidate itemsets containing subset of length k that are infrequent
    - Count the support of each candidate by scanning the DB
    - Eliminate candidates that are infrequent, leaving only those that are frequent

Support Counting: An Example

Suppose you have 15 candidate itemsets of length 3:

{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}

How many of these itemsets are supported by transaction (1,2,3,5,6)?



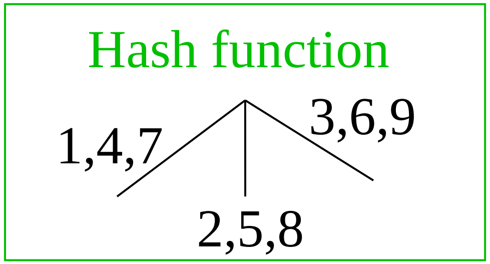
Support Counting Using a Hash Tree

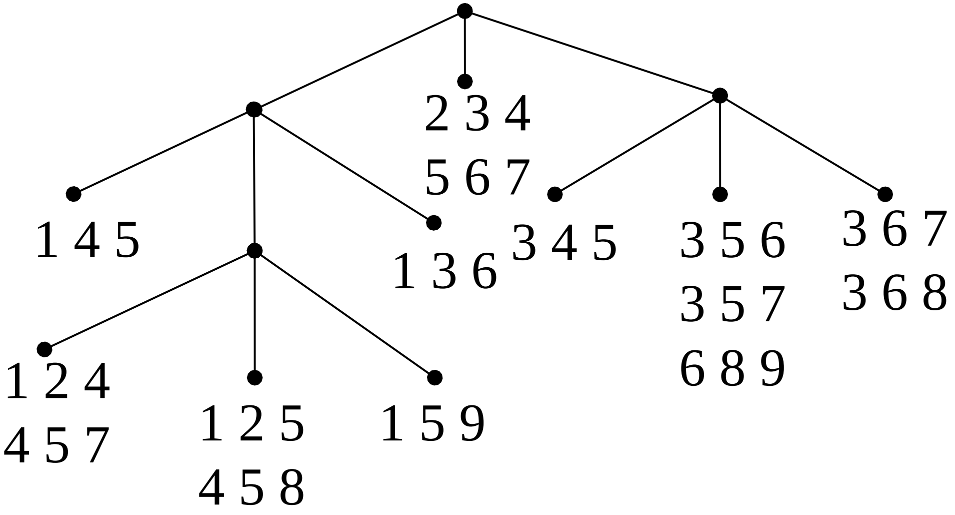
Suppose you have 15 candidate itemsets of length 3:

{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}

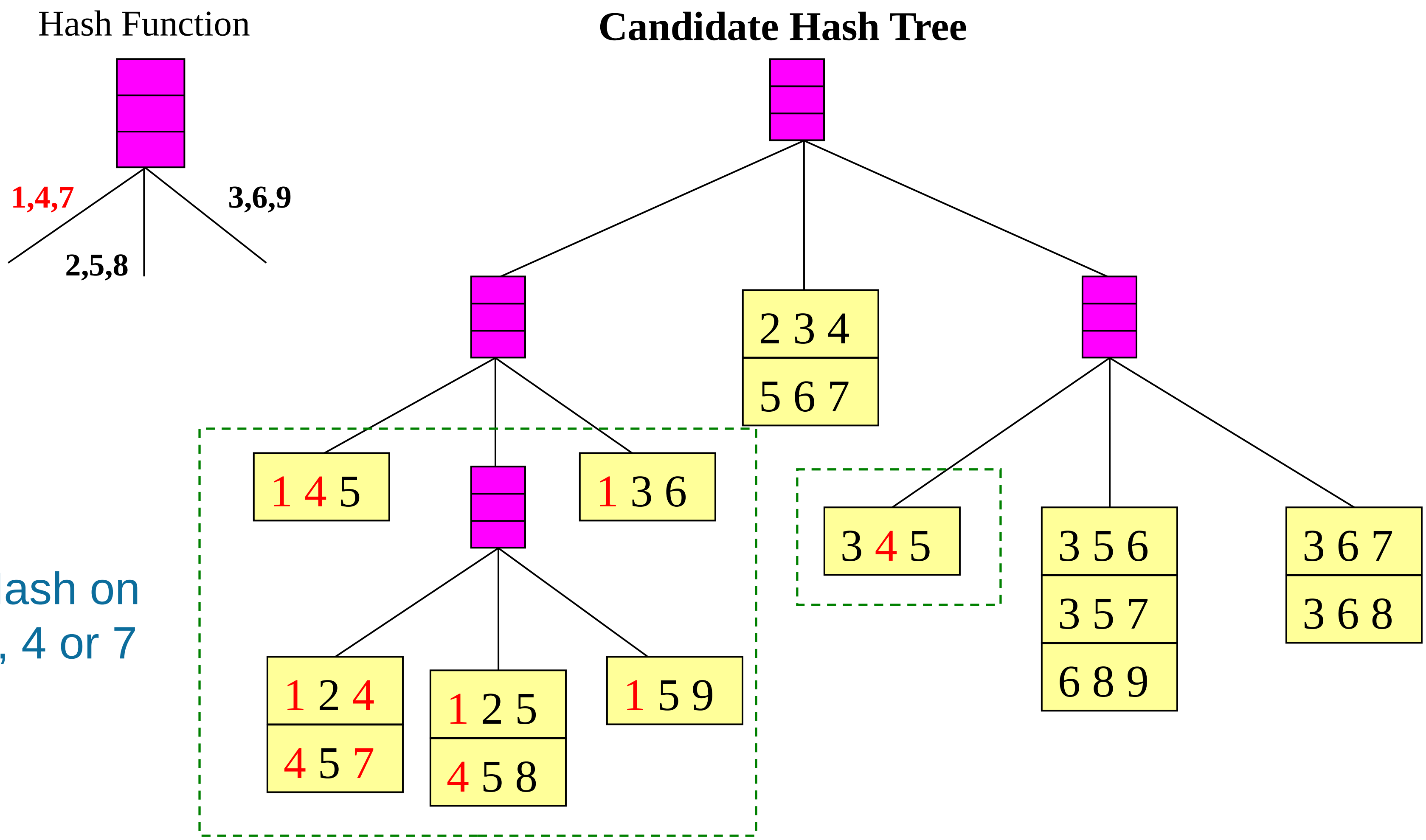
You need:

- **Hash function**
- **Max leaf size: max number of itemsets stored in a leaf node (if number of candidate itemsets exceeds max leaf size, split the node)**



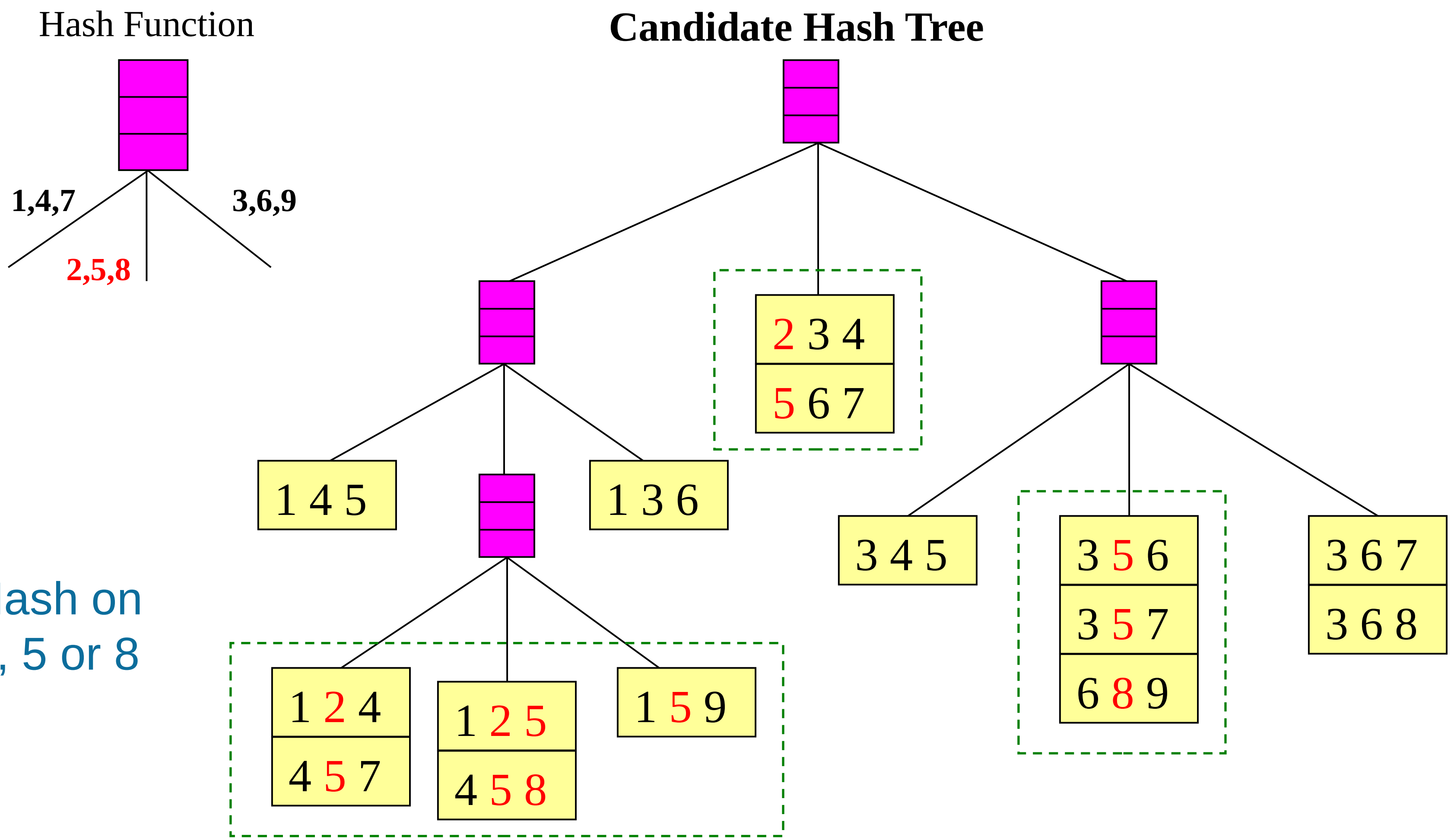


Support Counting Using a Hash Tree



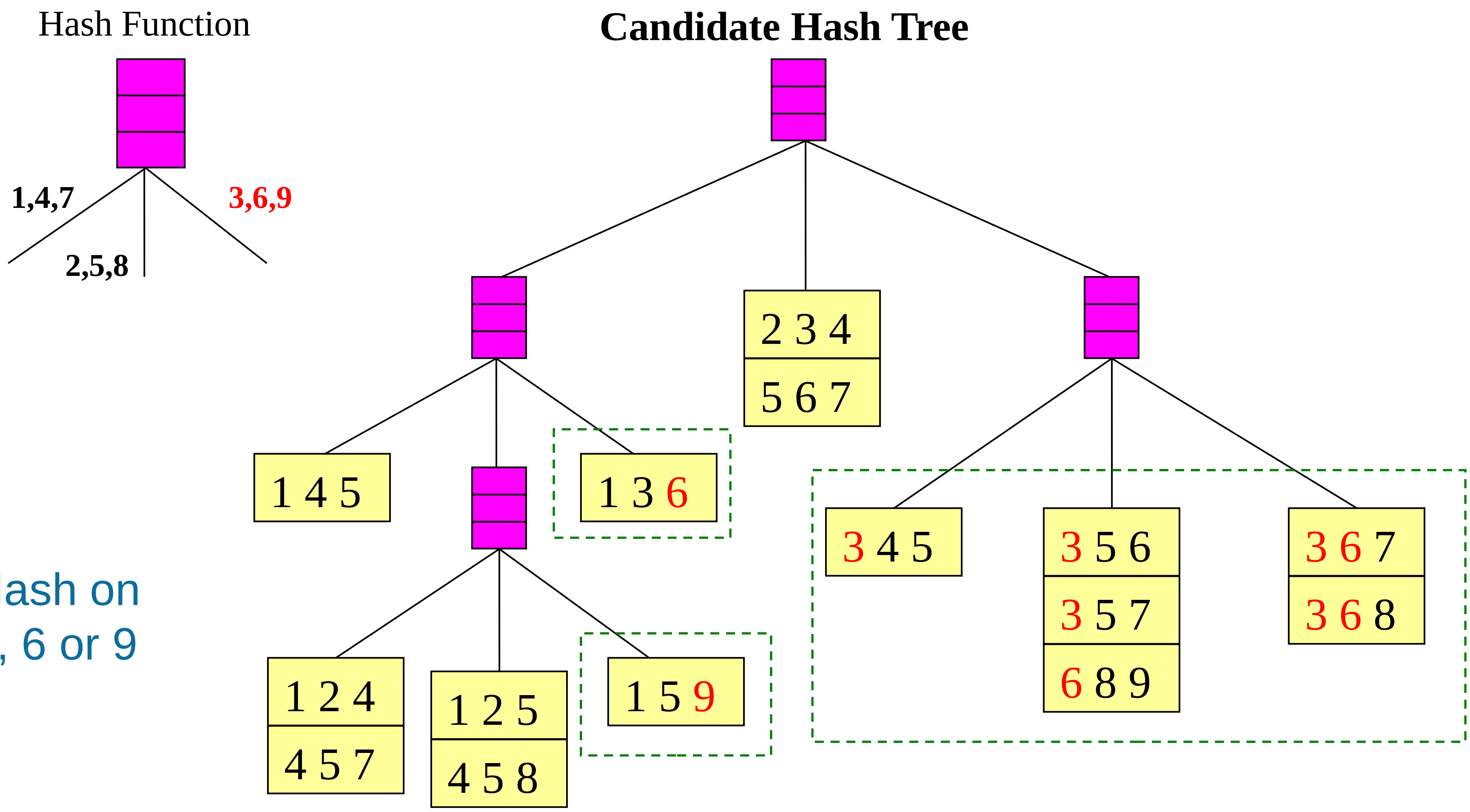
Hash on
1, 4 or 7

Support Counting Using a Hash Tree



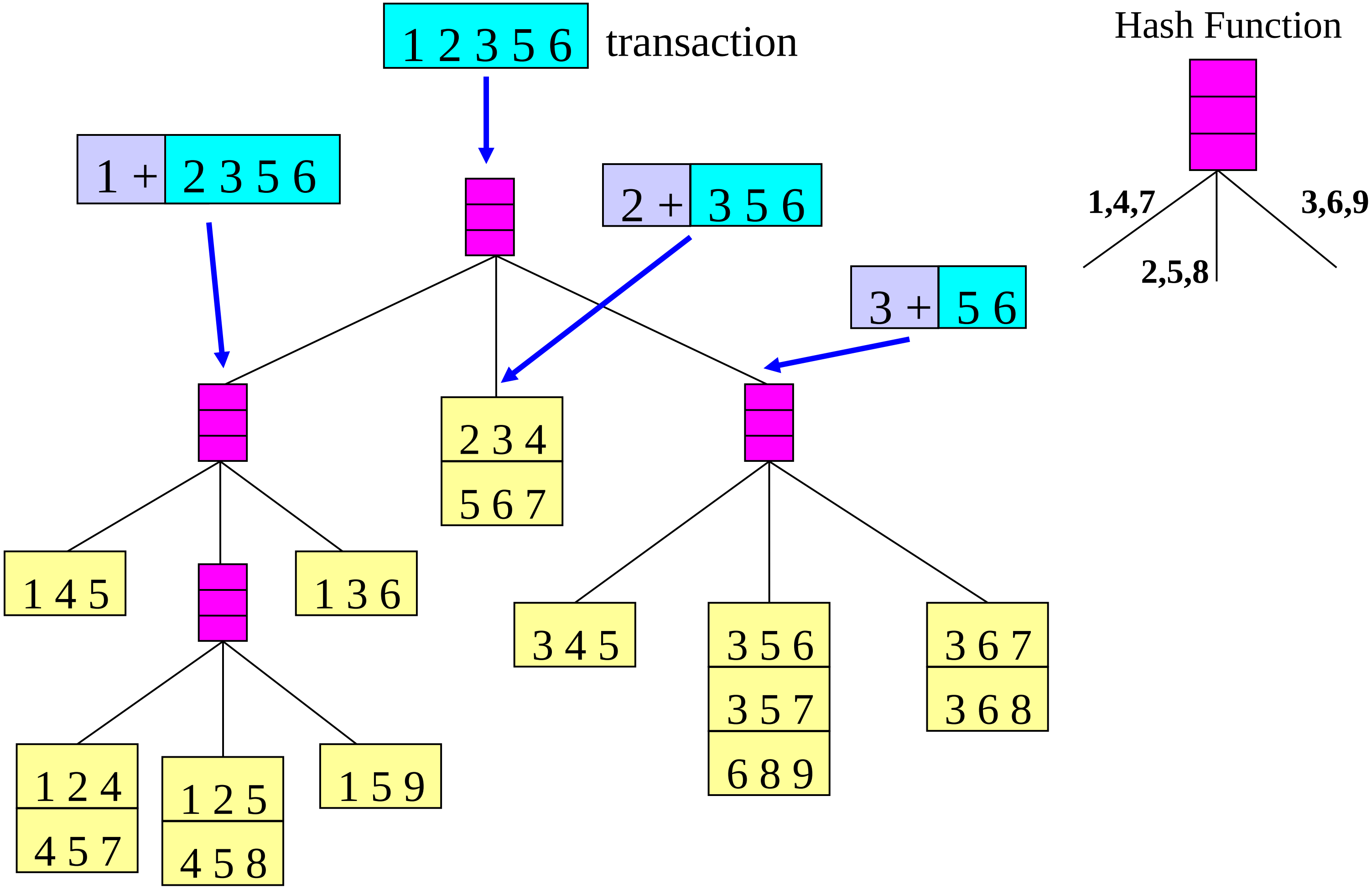
Hash on
2, 5 or 8

Support Counting Using a Hash Tree

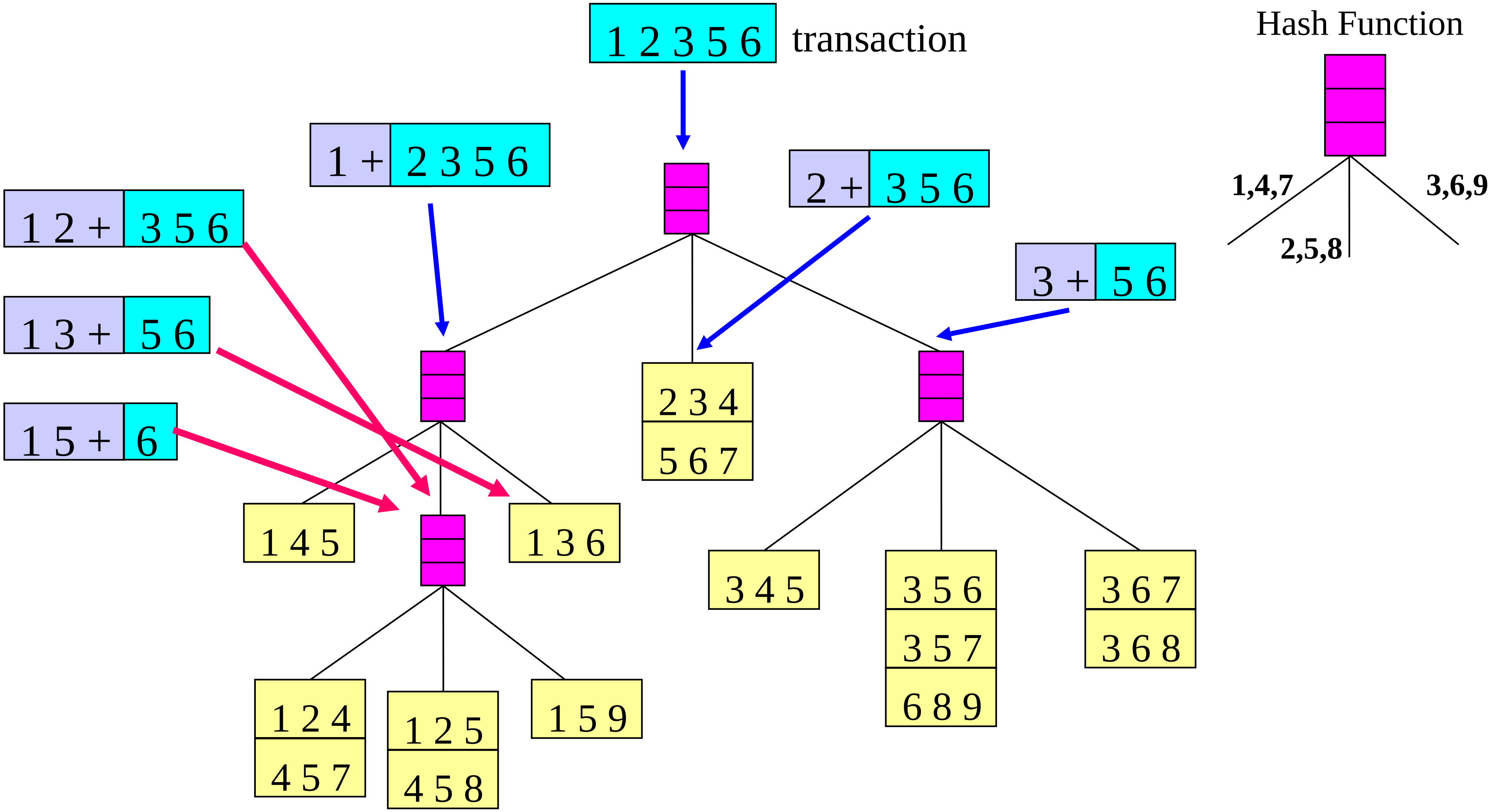


Hash on
3, 6 or 9

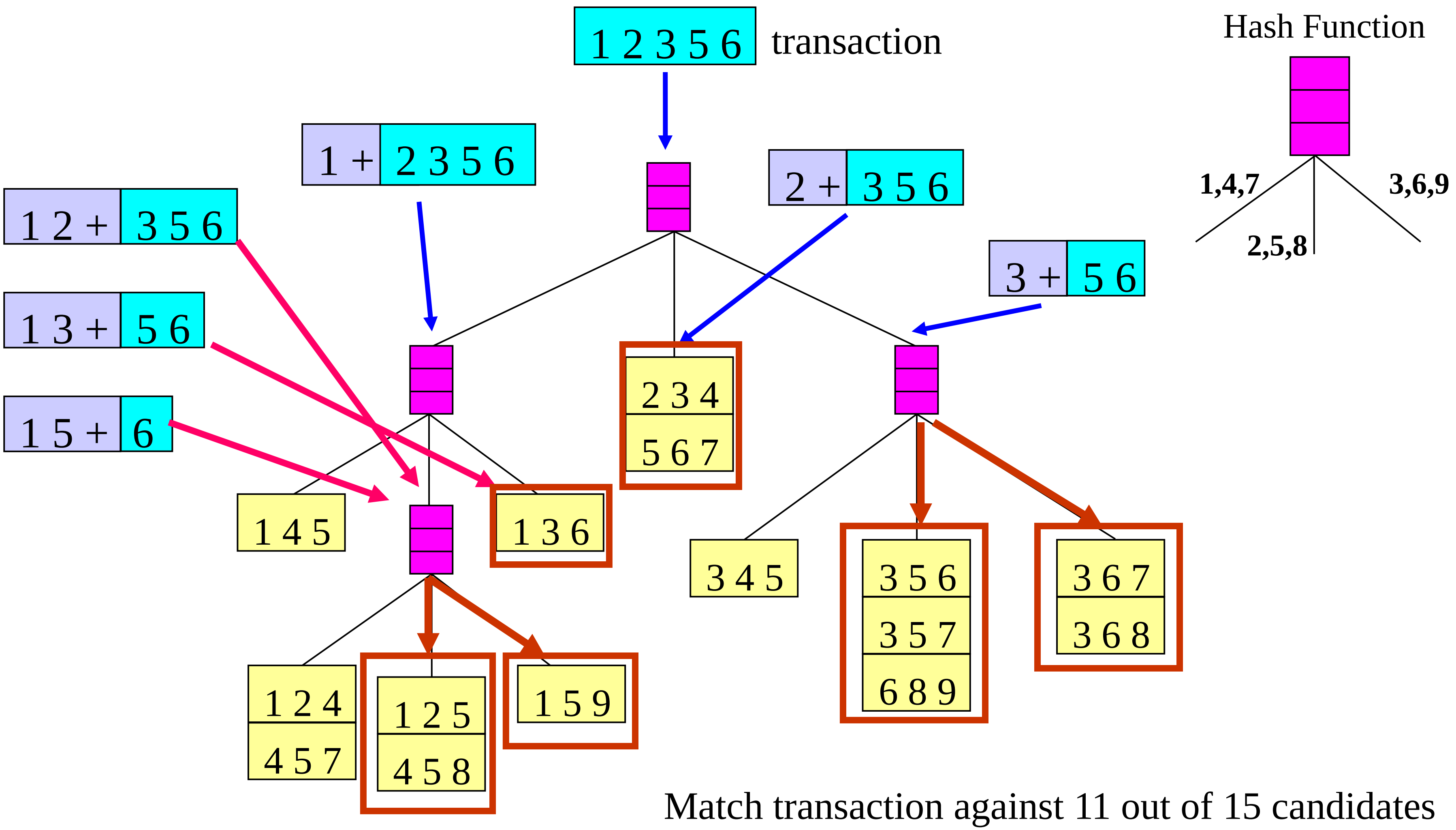
Support Counting Using a Hash Tree



Support Counting Using a Hash Tree



Support Counting Using a Hash Tree



Match transaction against 11 out of 15 candidates

Rule Generation

- Given a frequent itemset L, find all non-empty subsets f L such that f L – f satisfies the minimum confidence requirement
  - If {A,B,C,D} is a frequent itemset, candidate rules:

    ABC D, ABD C, ACD B, BCD A,
    A BCD, B ACD, C ABD, D ABC
    AB CD, AC BD, AD BC, BC AD,
    BD AC, CD AB,

- If |L| = k, then there are 2^k – 2 candidate association rules (ignoring L and L)

Rule Generation

- In general, confidence does not have an anti-monotone property

  c(ABC D) can be larger or smaller than c(AB D)

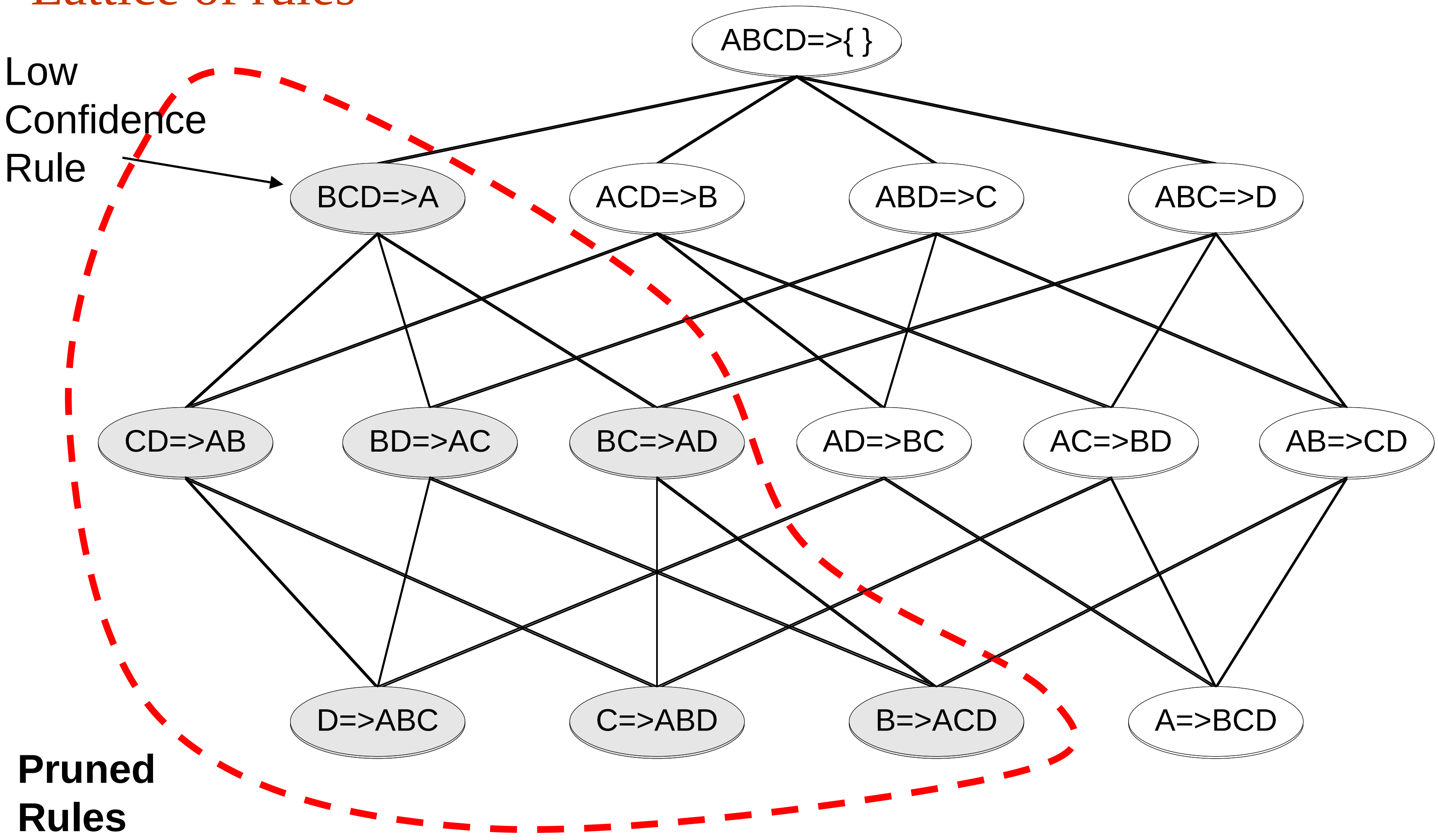
- But confidence of rules generated from the same itemset has an anti-monotone property
  - E.g., Suppose {A,B,C,D} is a frequent 4-itemset:

    c(ABC   D)   c(AB   CD)   c(A   BCD)

  - Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

Rule Generation for Apriori Algorithm

Lattice of rules



Association Analysis: Basic Concepts and Algorithms

Algorithms and Complexity

Factors Affecting Complexity of Apriori

- Choice of minimum support threshold

- Dimensionality (number of items) of the data set

- Size of database

- Average transaction width
  –

Factors Affecting Complexity of Apriori

- Choice of minimum support threshold
  - lowering support threshold results in more frequent itemsets
  - this may increase number of candidates and max length of frequent itemsets
- Dimensionality (number of items) of the data set
  - –
- Size of database
  - –
- Average transaction width
  - –

TID	*Items*
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Impact of Support Based Pruning

TID	*Items*
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Items (1-itemsets)

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

Minimum Support = 3

If every subset is considered,

$${}^6C_1 + {}^6C_2 + {}^6C_3$$

$$6 + 15 + 20 = 41$$

With support-based pruning,

$$6 + 6 + 4 = 16$$

Minimum Support = 2

If every subset is considered,

$${}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4$$

$$6 + 15 + 20 +15 = 56$$

Factors Affecting Complexity of Apriori

- Choice of minimum support threshold
  - lowering support threshold results in more frequent itemsets
  - this may increase number of candidates and max length of frequent itemsets
- Dimensionality (number of items) of the data set
  - More space is needed to store support count of itemsets
  - if number of frequent itemsets also increases, both computation and I/O costs may also increase
- Size of database
- Average transaction width
  -

TID	*Items*
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Factors Affecting Complexity of Apriori

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- Size of database
  - run time of algorithm increases with number of transactions
- Average transaction width

TID	*Items*
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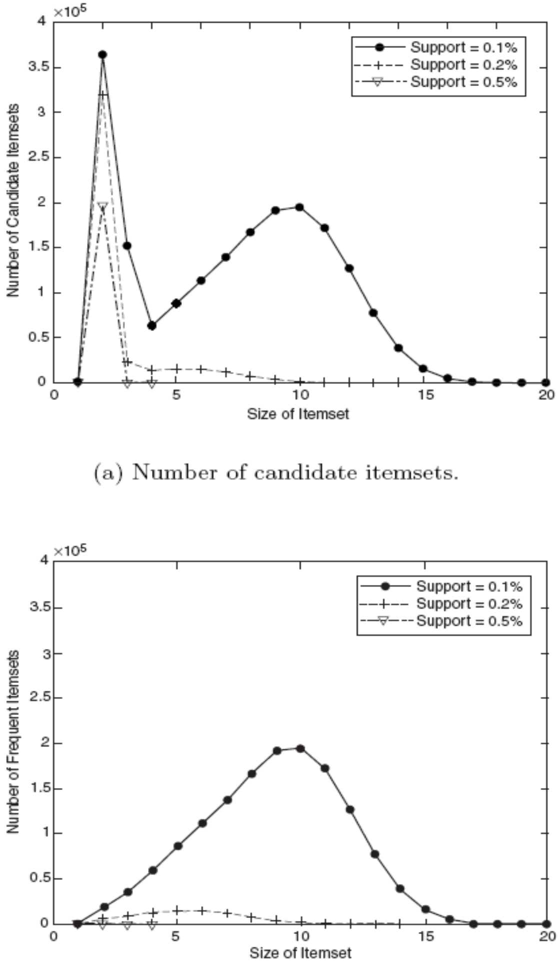
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Factors Affecting Complexity of Apriori

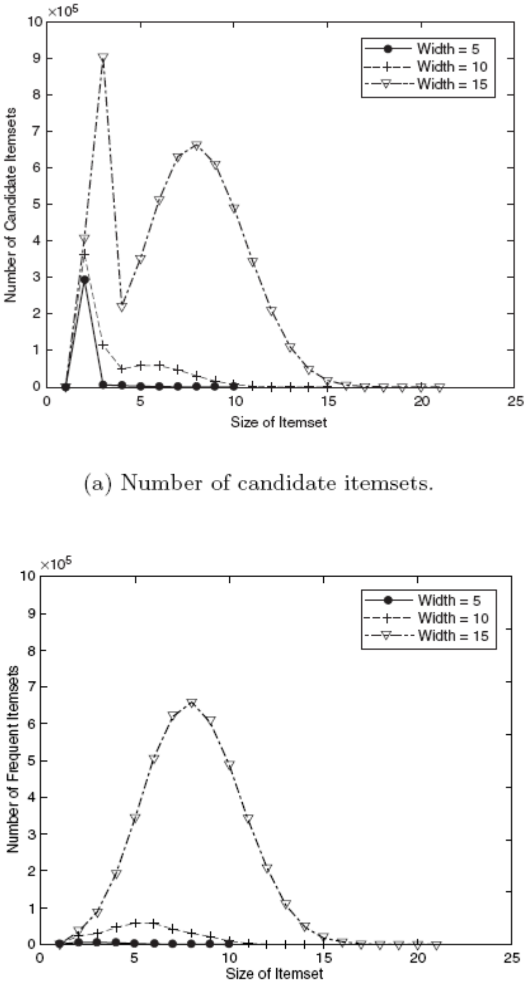
- Choice of minimum support threshold
  - lowering support threshold results in more frequent itemsets
  - this may increase number of candidates and max length of frequent itemsets
- Dimensionality (number of items) of the data set
  - More space is needed to store support count of itemsets
  - if number of frequent itemsets also increases, both computation and I/O costs may also increase
- Size of database
  - run time of algorithm increases with number of transactions
- Average transaction width
  - transaction width increases the max length of frequent itemsets
  - number of subsets in a transaction increases with its width, increasing computation time for support counting

Factors Affecting Complexity of Apriori



(a) Number of candidate itemsets.

(b) Number of frequent itemsets.



(a) Number of candidate itemsets.

(b) Number of Frequent Itemsets.

Figure 6.13. Effect of support threshold on the number of candidate and frequent itemsets.

Figure 6.14. Effect of average transaction width on the number of candidate and frequent itemsets.

Compact Representation of Frequent Itemsets

- Some frequent itemsets are redundant because their supersets are also frequent

Consider the following data set. Assume support threshold =5

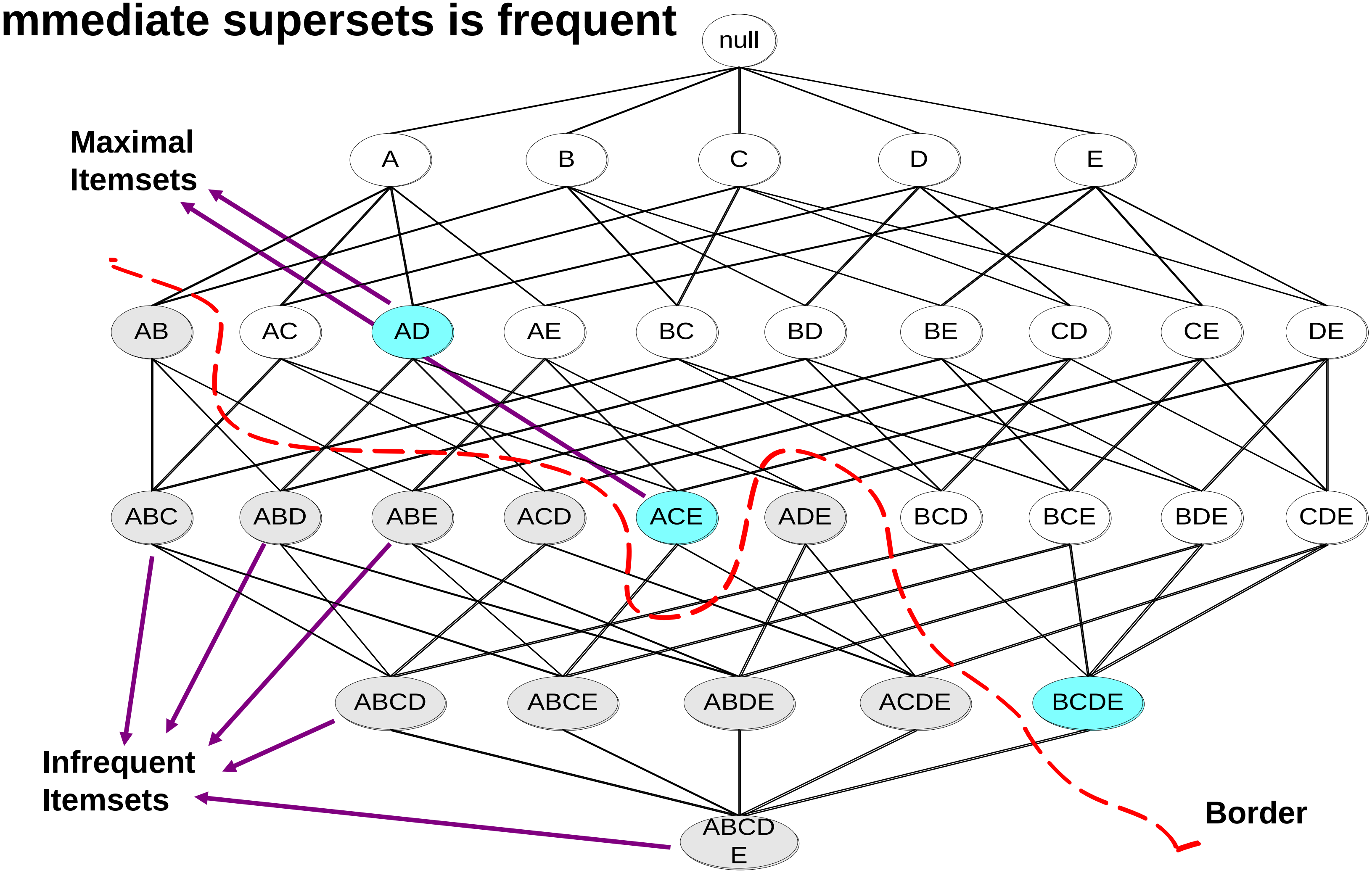
TID	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1

Number of frequent itemsets $= 3 \times \sum_{k=1}^{10} \binom{10}{k}$

- Need a compact representation

Maximal Frequent Itemset

An itemset is maximal frequent if it is frequent and none of its immediate supersets is frequent


null
A
B
C
D
E
AB
AC
AD
AE
BC
BD
BE
CD
CE
DE
ABC
ABD
ABE
ACD
ACE
ADE
BCD
BCE
BDE
CDE
ABCD
ABCE
ABDE
ACDE
BCDE
ABCD
E
Maximal
Itemsets
Infrequent
Itemsets
Border